What Would Arps Think About What We Have Done to His Decline Model?

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What Is the Arps Decline Model?

- Hyperbolic model is our focus

$$q(t) = \frac{q_i}{(1+bD_i t)^{\frac{1}{b}}}$$

where $0 < b < 1$

How Did Arps Come Up With the Model?

- Worked backward from two definitions
  - Decline rate, *D* (1/time)

$$D = -\frac{\frac{dq}{dt}}{q} = -\frac{d(lnq)}{dt}$$

  - Parameter *b* defined as change in 1/D (loss ratio) with time

$$b = \frac{d\left(\frac{1}{D}\right)}{dt}$$

- Key empirical observation: for most wells Arps/Cutler analyzed, *b* was constant (including *b* = 0) throughout all history

Arps' Hyperbolic Decline Model

- Integration for constant *b* leads to Arps' hyperbolic decline model

$$q(t) = \frac{q_i}{(1+bD_i t)^{\frac{1}{b}}}$$

- Implication: Arps hyperbolic decline equation valid only for *constant b*
- Hyperbolic model thoroughly validated (decades of successful application) for constant *b*, which requires BDF

Why Should Anyone Trust the Model?

- Arps' (and earlier investigators) finding (1944 and earlier) that the model
  - Fit most rate-time data well
  - Led to reasonable forecasts of future production
  - Rests on well-established empirical observation that *b* is reasonably constant in BDF

What Does the Model Require?

- Production at constant BHP
- Well or reservoir in boundary-dominated flow (BDF) (sometimes inappropriately called “pseudosteady-state flow”)
  - No transient flow data (oops!)
- Constant productivity index
  - No change in damage or stimulation
  - Skin factor constant
- Fixed drainage area
- For stabilized flow (BDF) with no change in productivity index, BHP, or drainage area, ‘*b* ’ should be constant for life of well

What Kind of Wells Did Arps Analyze?

- Key: Data that provided basis for model were from 1920’s, 1930’s, early 1940’s
  - All vertical wells
  - No hydraulic fracture stimulation (first in 1947)
  - Conventional permeability thus required for commerciality … generally, 10’s to 100’s of md

Why Does Permeability Matter?

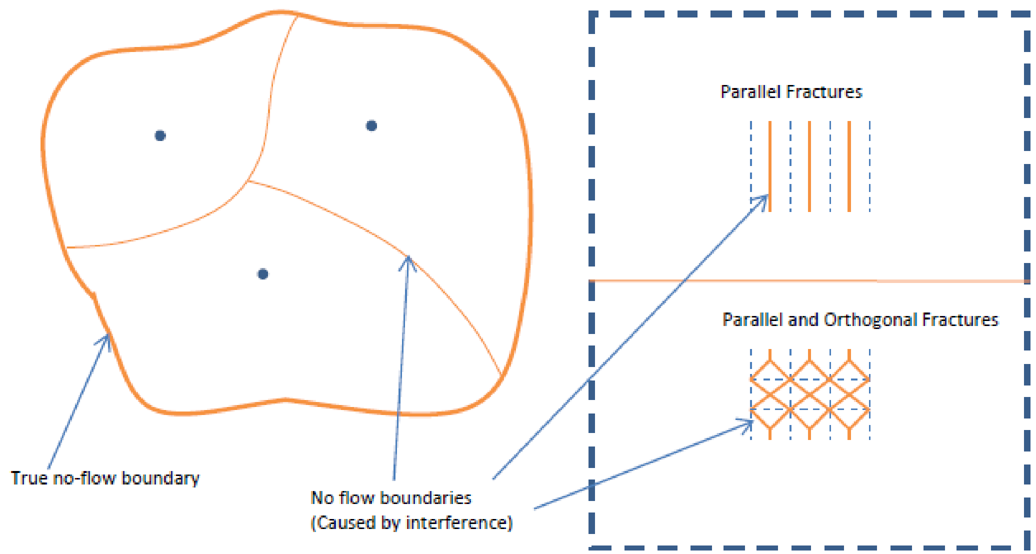
- Time to BDF in vertical wells estimated from

$$t_{bdf} = \frac{40\phi\mu c_t r_e^2}{k}$$

  - For 10 md gas reservoir, $t_{bdf} \approx$ 5 days for 160-acre spacing
  - For 100 nd gas reservoir, $t_{bdf} \approx$ 240 years for 160-acre spacing

… Arps' world and our world are different!

BDF Flow Caused Mostly by Interference



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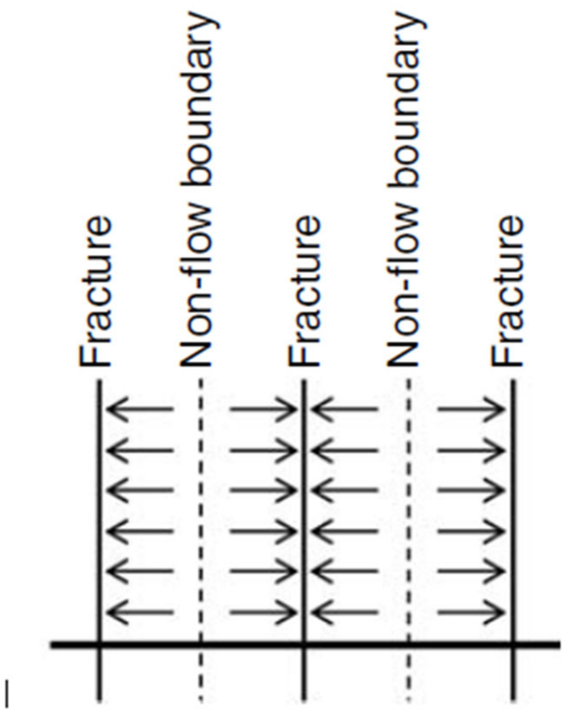
BDF Caused by Fracture Interference in Horizontal Wells with Multiple Fractures

- Even with close fracture stage spacing, time to BDF can be months or years in resource plays
- So: Can Arps' hyperbolic model work for modern wells in low permeability reservoirs?



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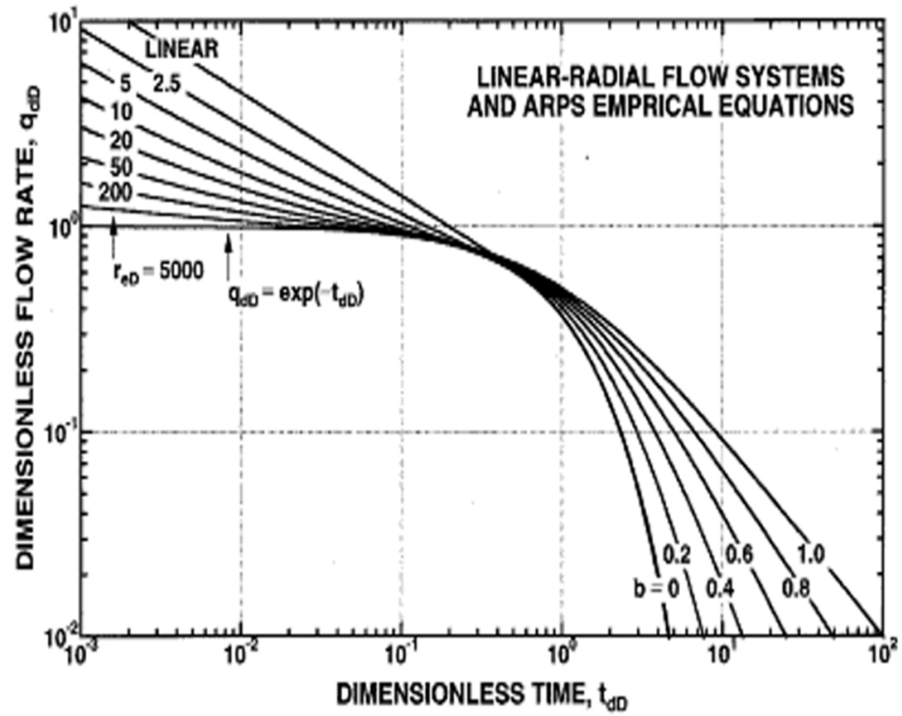
Arps Model in Resource Plays – Can it Work?

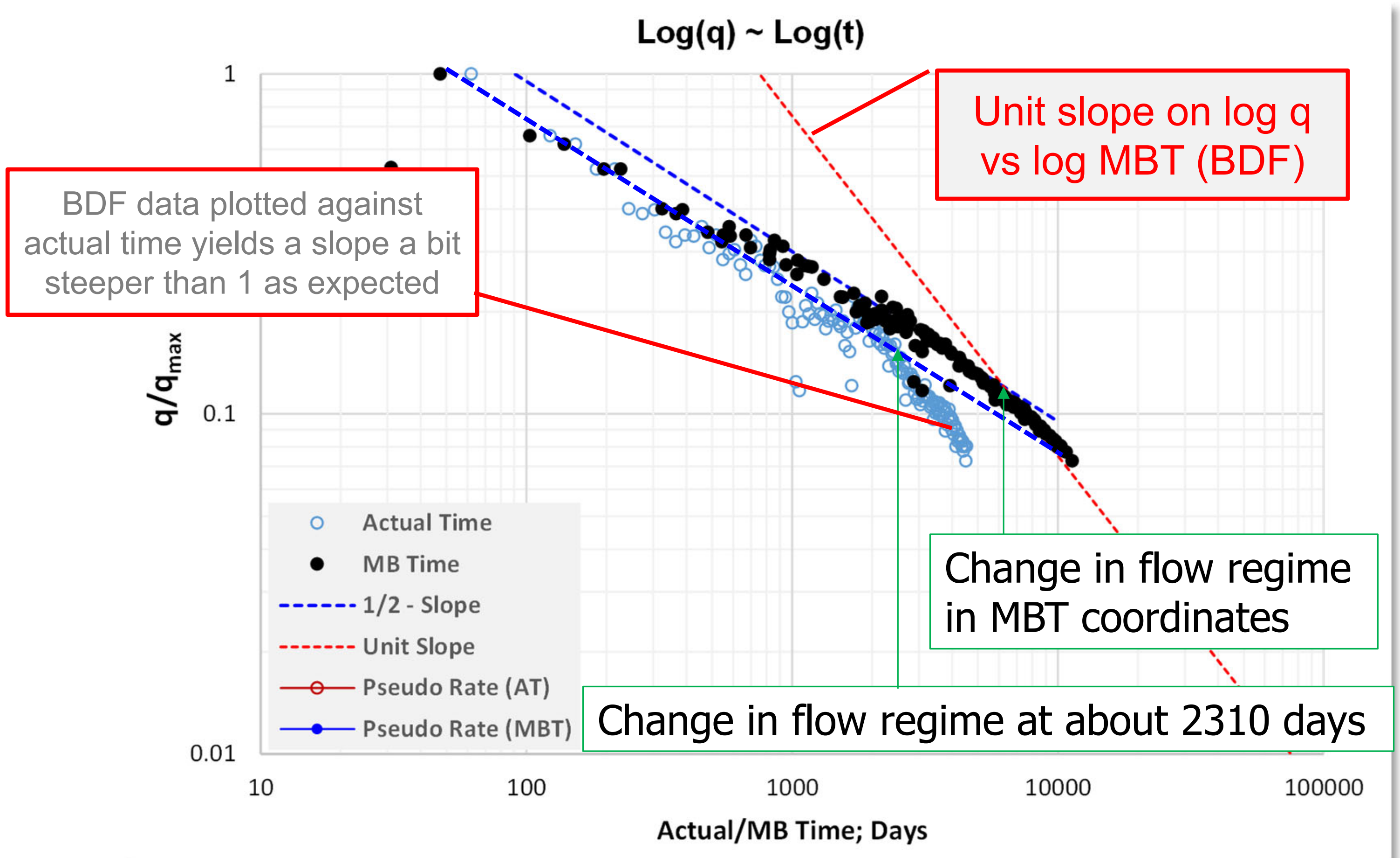
- Short answer: yes!
- How?
  - Divide flow history into multiple segments, apply model to each
    - Transient flow –often near linear flow
    - BDF – where Arps validated model
    - Transition region between transient and BDF
    - Early off-trend ramp-up period
  - How can we identify segments?
    - Log-log rate-time plots, assisted by rate-"material balance time" plots
    - Material-balance time: cumulative production/rate

But Does Data Fall on Straight Line ?

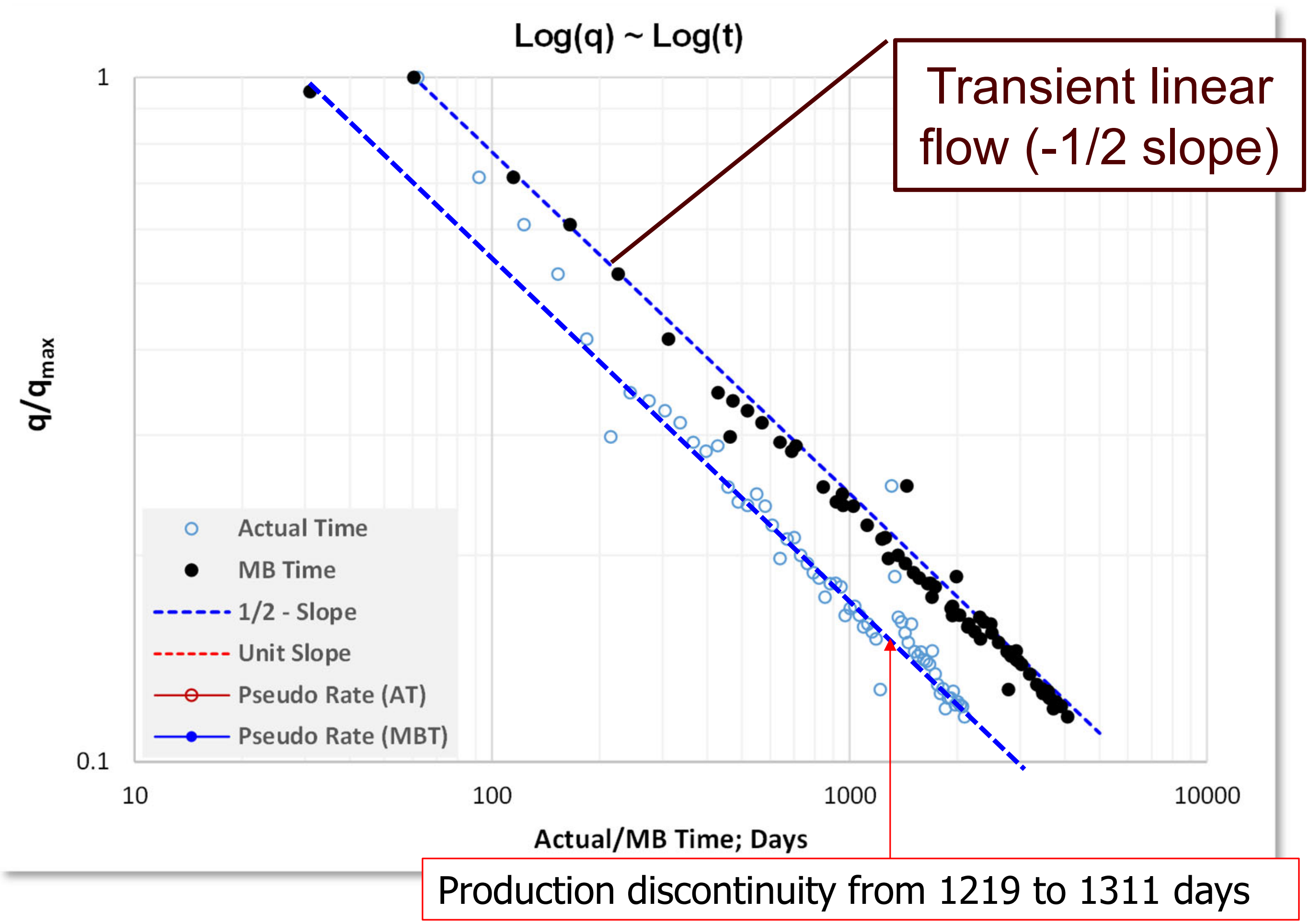
- Sometimes, yes
  - Early time "linear flow" $b = 2$
  - Also "power law" … straight line
  - BDF:
    - When b large, eventually
- Sometimes, no
  - BDF:
    - When b small, no



What Does Actual Field Data Look Like?



Do We Always See BDF?



Simple Method to Determine "*b*"

- Consider Arps hyperbolic decline model

$$q = \frac{q_i}{(1+bD_i t)^{\frac{1}{b}}}$$

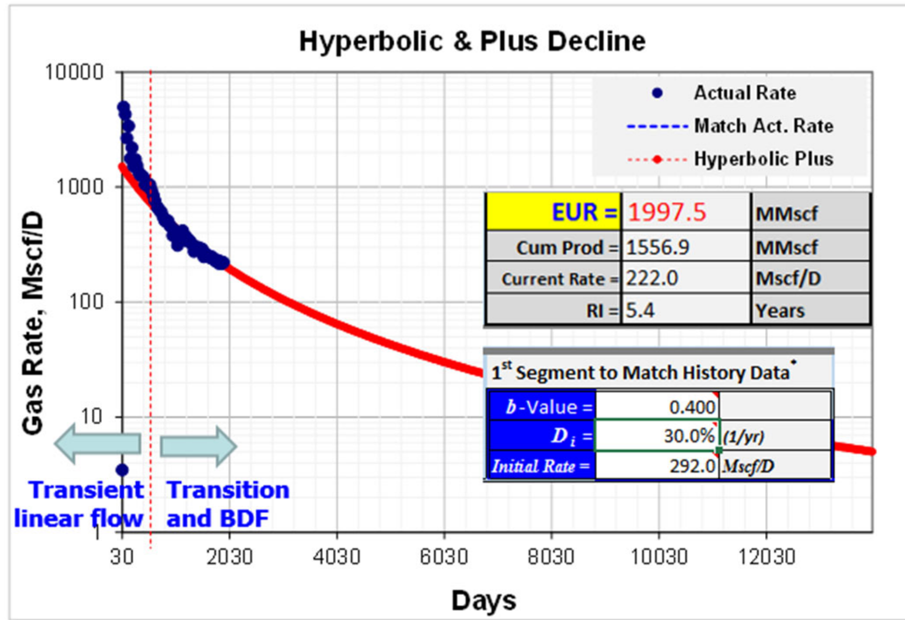
- Suppose $(bD_i t)$»1. Then, taking logs,

$$log(q) = log(q_i) - \left(\frac{1}{b}\right) log(bD_i t) = constant - \left(\frac{1}{b}\right) log(t)$$

- Conclusion: straight line on log-log *q* vs. *t* plot, slope = - $\left(\frac{1}{b}\right)$ … slick way to determine *b* ***if*** data fall on straight line
- Example: slope $= -\frac{1}{2} \rightarrow b = 2$ during transient flow

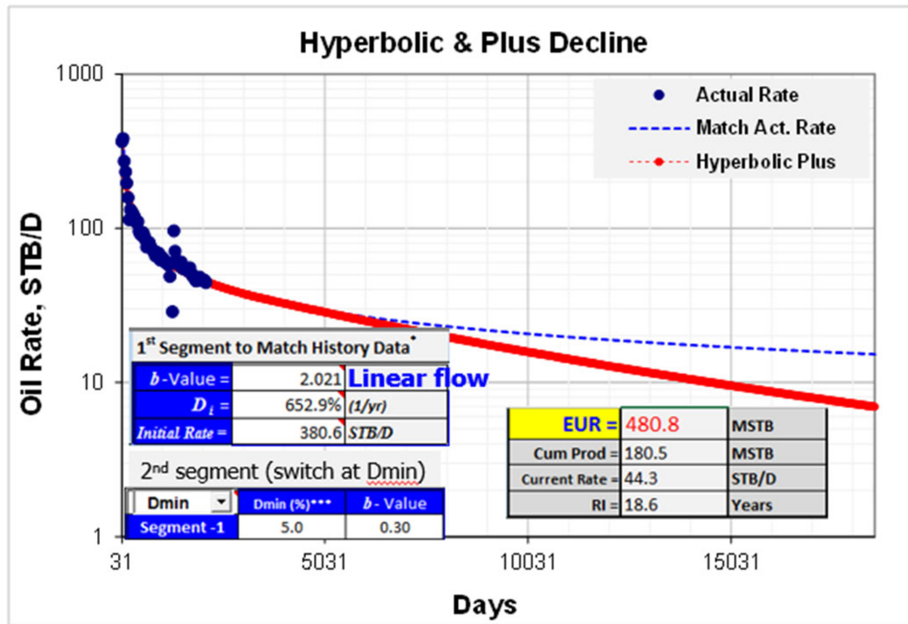
But How Can We Forecast with a Multi-Segment Arps Model?

- When we have some data in BDF, continue trend (same *b*) to life of well or economic limit rate
  - Assumption: interference with offset wells will have negligible impact



But What if We Have Only Transient Flow?

- Switch to transition model at specified D_{switch}, and switch to BDF model at specified D_{min} with specified b_{BDF}
- Decline rates at switch points and b_{BDF} from analogy or modeling



What Would Arps Think About This Procedure?

- Good reason to believe he would approve
  - Honors early ramp-up period, which he observed
  - Honors hyperbolic decline model he advocated during BDF
  - Adds transient flow regime present in modern low-*k* wells
    - When data lie on straight line on log-log plot, Arps hyperbolic model still appropriate (*b* constant)
    - b>1, but not for life of well … no laws of physics violated
  - Adds transition flow regime with varying *b*, but can use constant *b* (from analog) as first approximation

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